OLIMPIADA NAȚIONALĂ DE MATEMATICĂ

Faza locală-16.02.2019
Clasa a X-a
Bareme

1) Condiții de existență $x > \log_2 3$ (1)……………………………………………… ……………1p

Ecuația din enunț revine la $\lg\left(2^x-3\right)\cdot\left[\lg\left(2^x-3\right)-\lg 2\right]=1-\lg 2$ ………..………………..….2p

Notând cu $t=\lg\left(2^x-3\right)\Rightarrow t^2-t\cdot\lg 2-1+\lg 2=0$ ……………………………………........2p

Finalizare cu verificarea relației (1) $x_1=\log_2 13, x_2=4-\log_2 5$ ………………..………....2p

2) Condiții de existență $\begin{cases} x\geq 0 \\ 7-2\sqrt{x}-x>0 \end{cases} \Leftrightarrow x\in\left[0,9-4\sqrt{2}\right)\Rightarrow D=\left[0,9-4\sqrt{2}\right)$ … …..……..2p

$D\cap\mathbb{N}=\{0,1,2,3\}$, $f(0)=\log_{\sqrt{3}-1}7\notin\mathbb{N}$ …………………………………………………..1p

$f(1)=\log_{\sqrt{3}-1}4\notin\mathbb{N}$ ……………………………..……………………………………………1p

$f(2)=\log_{\sqrt{3}-1}\left(5-2\sqrt{2}\right)\notin\mathbb{N}$ ……………...1p

$f(3)=\log_{\sqrt{3}-1}\left(4-2\sqrt{3}\right)=\log_{\sqrt{3}-1}\left(\sqrt{3}-1\right)^2=2\in\mathbb{N}$, deci singurul punct cu coordonate naturale este $A(3,2)$ ……………………………………………………………….…………………...……2p

3) Scrise în formă trigonometrică $z_k=k(\cos\alpha_k+i\sin\alpha_k), \alpha_k\in[0,2\pi), k\in\{1,2,3,4\}$,…………2p

$\sum_{k=1}^{4}z_k=\sum_{k=1}^{4}k\cos\alpha_k+i\sum_{k=1}^{4}k\sin\alpha_k=10$ …………………………………..………..……....1p

$\sum_{k=1}^{4}k\cos\alpha_k=10, \sum_{k=1}^{4}k\sin\alpha_k=0\Rightarrow\alpha_1=\alpha_2=\alpha_3=\alpha_4=0$ …………………..…………………2p

$z_1=1, z_2=2, z_3=3, z_4=4\Rightarrow z_1+z_4=z_2+z_3$ ……………………………………………………2p

4) $z=\frac{\sqrt{3}}{2}\pm\frac{1}{2}i=\cos\frac{\pi}{6}\pm i\sin\frac{\pi}{6}\Rightarrow z^n+\frac{1}{z^n}=\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^n+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^n=2\cos\frac{n\pi}{6}$ ...2p

$n=6k\Rightarrow z^n+\frac{1}{z^n}=2(-1)^k$, $n=6k+1\Rightarrow z^n+\frac{1}{z^n}=2\cos\left(k\pi+\frac{\pi}{6}\right)=2(-1)^k\cos\frac{\pi}{6}=(-1)^k\sqrt{3}$

$n=6k+2\Rightarrow z^n+\frac{1}{z^n}=2\cos\left(k\pi+\frac{\pi}{3}\right)=2(-1)^k\cos\frac{\pi}{3}=(-1)^k$

$n=6k+3\Rightarrow z^n+\frac{1}{z^n}=2\cos\left(k\pi+\frac{\pi}{2}\right)=2(-1)^k\cos\frac{\pi}{2}=0$

$n=6k+4\Rightarrow z^n+\frac{1}{z^n}=2\cos\left(k\pi+\frac{2\pi}{3}\right)=2(-1)^k\cos\frac{2\pi}{3}=(-1)^{k+1}$

$n = 6k + 5 \Rightarrow z^n + \frac{1}{z^n} = 2\cos\left(k\pi + \frac{5\pi}{6}\right) = 2(-1)^k \cos\frac{5\pi}{6} = (-1)^{k+1}\sqrt{3}$ ……………………….3p

Deci $A = \left\{\pm 1, \pm 2, 0, \pm\sqrt{3}\right\} \Rightarrow \sum_{x \notin A} |x| = 6 + 2\sqrt{3}$

………………………………………………………….2p