OLIMPIADA NAŢIONALĂ DE MATEMATICĂ
Etapa locală 16.02.2019
Clasa a XI-a
Bareme de corectare şi notare

1. *Se consideră şirul* $(a_n)_{n>0}$ *cu termeni strict pozitivi. Dacă şirul* $(a_{n+1}-a_n)_{n>0}$ *este convergent către un număr real nenul, să se calculeze* $\lim\limits_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)^n$

BAREM :*Cum* $\lim\limits_{n\to\infty}(a_{n+1}-a_n)=\alpha\in\mathbb{R}_+^*$, *atunci* $\lim\limits_{n\to\infty} a_n=\infty$ *şi* $\lim\limits_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=1$,………………**2p**

$$\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)^n \overset{1^\infty}{=} \lim_{n\to\infty}\left(1+\frac{a_{n+1}}{a_n}-1\right)^n = \text{...}1\text{p}$$

$$\lim_{n\to\infty}\left(1+\frac{a_{n+1}-a_n}{a_n}\right)^n = \lim_{n\to\infty}\left(\left(1+\frac{\alpha}{a_n}\right)^{\frac{a_n}{\alpha}}\right)^{\frac{\alpha n}{a_n}} \text{...}2\text{p}$$

$$\overset{Stoltz}{=}\lim_{n\to\infty}\left(\left(1+\frac{\alpha}{a_n}\right)^{\frac{a_n}{\alpha}}\right)^{\frac{\alpha}{a_{n+1}-a_n}} = e\text{...}2\text{p}$$

2. *Fie matricea* $A\in M_3(\mathbb{Q})$ *ce satisface relaţia* $A^3=5A$. *Calculaţi rangul lui* A^{11}.

BAREM: $A^3=5A$. *De aici* $A^{11}=5^5A$ deci rang A^{11}= rang A....................................**2p**

Deasemenea din $(\det A)^3=5^3\det A$ *avem* $\det A=0$ *singura soluţie raţională*...................**2p**

Din relaţia $A^3-5A=O_3$ *-de fapt Hamilton-Cayley* $A^3-\mathrm{Tr}A\,A+d_2A-\det A\ I_3=O_3$, *determinanţii de ordinul 2 nu pot fi toţi nuli deci rangul lui A este* 2...**3p**

3.*Calculaţi* $\lim\limits_{x\to\infty}\frac{\sin x+\sin 2x+.........+\sin n^2x}{x+3x+....(2n-1)x}$, unde $n\geq 2, n$ natural

BAREM: $-1\leq\sin t\leq 1, \forall t\in\mathbb{R}$ ……………………………………………………..**1p**

Calculul sumei $x+3x+\dots\dots(2n-1)x = n^2x$…………………………………………….……**1p**

Deci $-\frac{n^2}{n^2x}\leq\frac{\sin x+\sin 2x+.........+\sin n^2x}{x+3x+....(2n-1)x}\leq\frac{n^2}{n^2x}$ ……………………….……….…..**3p**

şi prin criteriul cleştelui limita este 0……………………………**2p**

4.*Rezolvați în* $\mathbb{R}$ *ecuația :* $\begin{vmatrix} x & 0 & x \\ x & x^2 & -x \\ -x+a & -a^2 & x+a \end{vmatrix} = 0,$ unde $a \in \mathbb{R}$

BAREM:

$$\begin{vmatrix} x & 0 & x \\ x & x^2 & -x \\ -x+a & -a^2 & x+a \end{vmatrix} = 0 \underset{L_3+L_2}{\overset{L_1+L_2+L_3}{\Longrightarrow}} \begin{vmatrix} x+a & x^2-a & x+a \\ x & x^2 & -x \\ a & x^2-a^2 & a \end{vmatrix} = 0 \text{............................2p}$$

$$\Rightarrow (x+a)\begin{vmatrix} 1 & x-a & 1 \\ x & x^2 & -x \\ a & x^2-a^2 & a \end{vmatrix} = 0 \text{..1p}$$

$$\underset{C_1-C_3}{\Longrightarrow} (x+a)\begin{vmatrix} 0 & x-a & 1 \\ 2x & x^2 & -x \\ 0 & x^2-a^2 & a \end{vmatrix} = 0 \text{..2p}$$

prin dezvoltare avem: $2x(x+a)(x^2-ax)=0,$ *cu solutiile* $0, \pm a$..................2p

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